CERN-TH/98-43
hep-th/9802057

A SIMPLE/SHORT INTRODUCTION TO PRE-BIG-BANG PHYSICS/COSMOLOGY

GABRIELE VENEZIANO
Theory Division, CERN,
1211 Geneva 23, Switzerland

ABSTRACT

A simple, non-technical introduction to the pre-big bang scenario is given, emphasizing physical motivations, considerations, and consequences over formalism.

1. Introduction

It is commonly believed (see e.g.[1]) that the Universe – and time itself – started some 15 billion years ago from some kind of primordial explosion, the famous Big Bang. Indeed, the experimental observations of the red-shift and of the Cosmic Microwave Background (CMB) lead us quite unequivocally to the conclusion that, as we trace back our history, we encounter epochs of increasingly high temperature, energy density, and curvature. However, as we arrive close to the singularity, our classical equations are known to break down. The earliest time we can think about classically is certainly larger than the so-called Planck time, $t_P = \sqrt{G_N \hbar} \sim 10^{-43}$s ($c = 1$ throughout). Hence, the honest answer to the question: Did the Universe and time have a beginning? is: We do not know, since the answer lies in the unexplored domain of quantum gravity.

Besides the initial singularity problem – and in spite of its successes – the hot big bang model also has considerable phenomenological problems. Amusingly, these too can be traced back to the nature of the very early state of the Universe. Let us briefly recall why.

General Relativity, together with the cosmological principle (i.e. the assumption of a homogeneous, isotropic Universe over large scales), allows us to describe the geometry of space-time through the Friedmann-Robertson-Walker (FRW) metric (see e.g.[2]):

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] , \; d\Omega^2 = d\theta^2 + \sin^2\theta \mathrm{d}\varphi^2 \; , \qquad (1)$$

where, as usual, the discrete variable $k = 0, 1, -1$ distinguishes the cases of a flat, closed, or open Universe, respectively. In the presence of some matter (fluid), described by an energy density ρ and pressure density p, the evolution of the Universe is controlled by the Einstein-Friedmann equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$
$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \ , \tag{2}$$

which, together, imply the energy conservation equation:

$$\dot{\rho} = -3\left(\frac{\dot{a}}{a}\right)(\rho + p) \ . \tag{3}$$

Given a model for the sources, or, more specifically, given a relation between ρ and p (so-called equation of state), one can easily solve these equations and find the usual FRW cosmological solutions. For the matter- and radiation-dominated cases, respectively,

$$p = 0 \ \Rightarrow \ a(t) \sim t^{2/3}, \ \rho \sim a^{-3}$$
$$p = \rho/3 \ \Rightarrow \ a(t) \sim t^{1/2}, \ \rho \sim a^{-4} \tag{4}$$

where, for simplicity, the spatial-curvature term $\frac{k}{a^2}$ was neglected.

The so-called horizon problem arises as follows. The observable part of our Universe, our present horizon, is given by the distance that light has travelled since the big bang, or about 10^{28} cm. At earlier times, the horizon was much smaller. For a hypothetical observer looking at the sky a few Planck times after the Big Bang, the horizon was not much bigger that a few Planck lengths, say about 10^{-32} cm. Instead, as easily seen from (4), the portion of space that corresponds to our present horizon was, for that same hypothetical observer, some 30 orders of magnitude larger than the Planck length, or about 1 mm. In other words, at that time, what has nowadays become our observable Universe consisted of $(10^{30})^3 = 10^{90}$ Planckian-size, causally disconnected regions. There is no reason to expect that conditions in all those regions were initially the same, since there was never any thermal contact between them. Yet, today, all those 10^{90} regions make up our observable Universe and all appear to resemble one another (to within one part in 10^5).

Who prepared such a smooth and very unlikely initial state? Perhaps God, who picked up a very special point in the huge space of all possible initial configurations (see, in this connection, a nice picture in Roger Penrose's book[3]). If, instead, we do not accept God's fine-tuning, or, in more scientific terms, we do not want to attribute homogeneity to some unknown Planckian physics, two logical possibilities are left to our choice:

- Time did have its beginning at the big bang, when initial conditions were rather random, but a period of superluminal expansion (inflation) brought all those 10^{90} patches in causal contact sometimes between the big bang and the present time. This is the standard *post-big bang* inflation paradigm (see e.g.[4]).

- Time did *not* have its beginning with the big bang and some pre-big bang physics cooked up a "good" big bang from a more generic (less fine-tuned) initial state. This is the attitude one takes in the so-called *pre-big bang* scenario.

One can similarly argue that there are two ways of solving the second major problem of standard cosmology, the so-called flatness problem. Today, space is, to a very good approximation, Euclidean. If it does have any spatial curvature (represented by the $\frac{k}{a^2}$ term in the cosmological equations), this is of $O(H^2)$, i.e. extremely small. Given the solutions (4) it is easy to check that, in order to have such conditions today, one has to start, at the Planck time, with an extremely flat Universe, i.e. with a curvature radius a some 30 orders of magnitude larger that the characteristic length scale at the time, $H^{-1} \sim l_P$. Again, two possibilities come to mind: either the Universe was not particularly flat at the beginning – and subsequent inflation stretched out spatial curvature – or some pre-big bang physics prepared a nice spatially-flat big bang.

Conventional inflation again chooses the first alternative. To succeed, it needs a weakly-coupled scalar field, the so-called inflaton, which, some time after the big bang, finds itself *homogeneously* displaced from the minimum of its potential, and slowly rolls towards it. While doing so, if certain conditions are met, the effective equation of state is $p \sim -\rho$ and the Universe expands quasi-exponentially. One needs this period of exponential growth to last for a long enough time for all our accessible Universe to come in causal contact. This can be achieved at the price of fine-tuning certain masses or couplings.

My main reservations towards this solution (see[5] for a different criticism) are that no one has a convincing model for what the inflaton ought to be and, even more seriously, that it is not easy to justify the initial conditions that can provide a sufficiently long inflationary phase. One is back somehow to the starting point, since the conditions at the onset of inflation have to come from a previous phase, and this inevitably leads us to giving initial conditions in the mysterious Planckian era. Quantum cosmology has then been invoked; however, I am also sceptical about present quantum cosmology arguments[6] "predicting" inflation since they are based on the so-called minisuperspace truncation of the Wheeler–DeWitt equation. Such an approach is only justified for a fairly homogeneous initial universe, which is just what we do *not* wish to assume.

How come string theory prefers the second way out? In order to explain this I have to open a parenthesis and tell you about some striking properties of quantum (super)strings.

2. Three properties of (super)strings

I will concentrate on just three properties of strings which are crucial to the understanding of their possible cosmological implications:

- 1. There is a fundamental length scale in string theory, providing a characteristic size for strings and thus an ultraviolet cutoff. Thanks to this property, superstring theory can be taken seriously as a candidate finite quantum theory of gravity (and of the other interactions as well). The fundamental length scale λ_s is given in terms of the string tension T by the formula

$$\lambda_s = \sqrt{\hbar/T} \ . \tag{5}$$

Actually λ_s, rather than T, is the fundamental parameter of the theory, providing a meaning for what short and large distances mean. When fields vary little over a string length λ_s one recovers a field-theoretic description given by a local Lagrangian with the smallest number of derivatives.

- 2. Couplings are not God-given constants; they are VEVs which, hopefully, become dynamically determined. In particular, a scalar field, the so-called dilaton ϕ, controls all sorts of couplings, gravitational and gauge alike. Since, in our normalizations,

$$T \cdot G_N = l_P^2/\lambda_s^2 \sim \alpha_{gauge} \sim e^{\phi} \ , \tag{6}$$

we see that the weak coupling region is $\phi \ll -1$. By contrast, at present, $e^{\phi} \sim 1/25$, implying $\lambda_s \sim 10 l_P \sim 10^{-32}$ cm. In the weak coupling region, perturbative superstring theory is an adequate description of physics, implying that the dilaton itself behaves like a massless particle. As such, the dilaton can easily evolve cosmologically while it is deeply inside the perturbative region. Precision tests of the equivalence principle imply, however, that the dilaton has a mass, i.e. that near its present value $\phi = O(-1)$, its potential has a minimum with finite curvature.

If both the coupling and derivatives are small, physics is adequately described by the tree-level low-energy effective action of string theory, which reads:

$$\begin{aligned} \Gamma_{eff} &= \frac{1}{2}\int d^4x\sqrt{-g}\ e^{-\phi}\left[\lambda_s^{-2}(\mathcal{R} + \partial_\mu\phi\partial^\mu\phi + H_{\mu\nu\rho}^2) + F_{\mu\nu}^2 + \text{higher derivatives}\right] \\ &+ \left[\text{higher orders in } e^{\phi}\right] \ , \end{aligned} \tag{7}$$

where we have included the contributions of the Kalb-Ramond antisymmetric tensor field through its field strength $H_{\mu\nu\rho}$. Note the two kinds of corrections alluded to in (7). They intervene, respectively, whenever space-time derivatives (i.e. energies) or the string coupling e^{ϕ} become appreciable. Equation (7) will be our starting point to describe pre-big bang cosmology.

- 3. Cosmological field equations exhibit new stringy symmetries whose most interesting representative (scale-factor duality or SFD) acts as follows:

$$a(t) \to a^{-1}(-t) \ , \ \phi(t) \to \phi(-t) - 2d \ \log a(-t) \ , d = 3 \ . \tag{8}$$

The interest of this duality transformation lies in the fact that it maps ordinary FRW cosmologies with $H > 0$, $\dot{H} < 0$, and $\dot{\phi} = 0$ at $t > 0$ into inflationary cosmologies with $H > 0$, $\dot{H} > 0$, and $\dot{\phi} > 0$ at $t < 0$. Actually, the dual cosmologies are of the so-called super-inflation (or pole-inflation) type, i.e. have a growing – rather than a constant – Hubble parameter. Since many of the distinctive consequences of PBB cosmology originate from this peculiar feature, let me explain in simplified terms where it comes from Eqs. (2) imply that, for an expanding Universe, ρ and H^2, being proportional (take for simplicity the case of $k = 0$), decrease together in time. This is true if G_N is constant. In string theory, where G_N is controlled by the dilaton through Eq. (6), it is perfectly possible to have a growing H while ρ decreases, provided ϕ is also growing.

The suggestion from string theory now becomes almost a compelling one: Can one put together a standard cosmology at $t > 0$ and a dual cosmology at $t < 0$ to generate a single scenario containing dilaton-driven pre-big bang inflation and FRW post-big bang behaviour? Since, in such a construct, the Hubble parameter grows for $t < 0$ and decreases for $t > 0$, it should reach its maximum at $t = 0$, instant therefore identified with the occurrence of the Big Bang of standard cosmology. The problem is that this maximum is actually infinite if one works in the context of the low-energy effective theory, i.e. if all corrections in (7) are neglected. The pre-(post-) big bang phases have singularities in the future(past), probably a consequence of the validity, in that approximation, of the assumption leading to the Hawking-Penrose singularity theorems.[7] However, that approximation breaks down as soon as the Hubble parameter reaches values $O(\lambda_s^{-1})$ leading us to expect that the maximal value of H, reached at $t = 0$, should be actually of the order of the fundamental length of string theory.

It is easy, actually, to write down mathematical expressions that interpolate smoothly between the inflationary and the FRW branches. For instance, the ansatz

$$\begin{aligned} \hat{a}(t) &= \left(\frac{t + \sqrt{t^2 + \lambda_s^2}}{\lambda_s}\right)^{1/2} \\ \hat{\phi}(t) &= \phi_0 + \frac{3}{2} \log\left(1 + \frac{t}{\sqrt{t^2 + \lambda_s^2}}\right) \end{aligned} \qquad (9)$$

is easily checked to approach a standard radiation-dominated cosmology with constant dilaton at $t \gg \lambda_s$ and to a dual dilaton-driven inflationary cosmology at $t \ll -\lambda_s$. The question is: Does anything like this come from the true (i.e. high-curvature and/or loop-corrected) field equations? Leaving this hard question to the final section, we turn instead to the formulation of the basic pre-big bang postulate.

3. The pre-big bang postulate

Clearly, if we want to use a dual cosmology for the prehistory of the Universe, given the positive signs of $\dot{H}$ and $\dot{\phi}$, we have to start from (very?) small initial values for H and e^ϕ. Although not strictly necessary, we will also impose, for the sake of simplicity, an almost empty initial Universe. This leads us to the following basic postulate of PBB cosmology:

The Universe started its evolution from the most simple initial state conceivable in string theory, its perturbative vacuum. This corresponds to an (almost)

EMPTY, COLD, FLAT, FREE

Universe as opposed to the standard

DENSE, HOT, HIGHLY-CURVED

initial state of conventional cosmology.

For this assumption to make sense I will have to argue that the new initial conditions are able to provide, at later times, a hot big bang with the desired characteristics thanks to a long pre-big bang inflationary phase. This looks a priori a very hard task, but I will explain below how it can possibly happen. Before discussing this let me illustrate, in two figures, the qualitative differences between the standard (non-inflationary) model, the standard inflationary scenario, and ours.

In Fig. 1 I am plotting, against cosmic time, the behaviour of the Hubble parameter H measured in Planck units $M_P \sim 10^{19}$ GeV. Note that, while in the first model H is a concave function of time, in the second it has a long flat plateau where $H/M_P \leq 10^{-5}$ (this constraint comes from COBE's data, see e.g.[4]). Finally, in the proposed scenario, H/M_P grows all the way to a maximal value $O(10^{-1})$, but quickly becomes very small at both large positive and large negative t.

In Fig. 2 we can see the consequences of this behaviour of H on the kinematics of horizon crossing. During inflation, increasingly small scales are pushed out of the horizon by the accelerated expansion of the Universe. However, while in standard inflation (Fig. 2a) larger scales cross the horizon at slightly larger values of H, in the pre-big bang scenario (Fig. 2b) it is the other way around: the larger the scale, the smaller the value of H at horizon crossing. As we shall see in Sec. 4, the value of H/M_P at horizon crossing is the determining quantity for evaluating the present magnitude of quantum fluctuations at different length scales. Hence the above kinematics of horizon crossing will have an important bearing on the spectrum of quantum fluctuations.

Before turning to that, we should discuss how dilaton-driven inflation sets in during the pre-big bang phase.

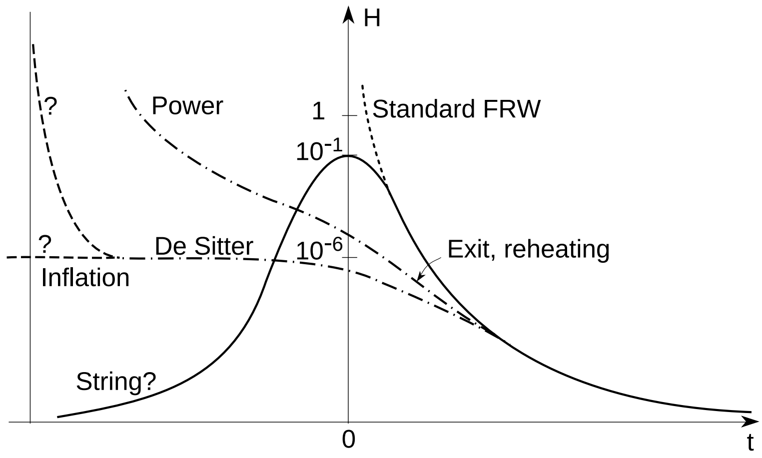

Figure 1

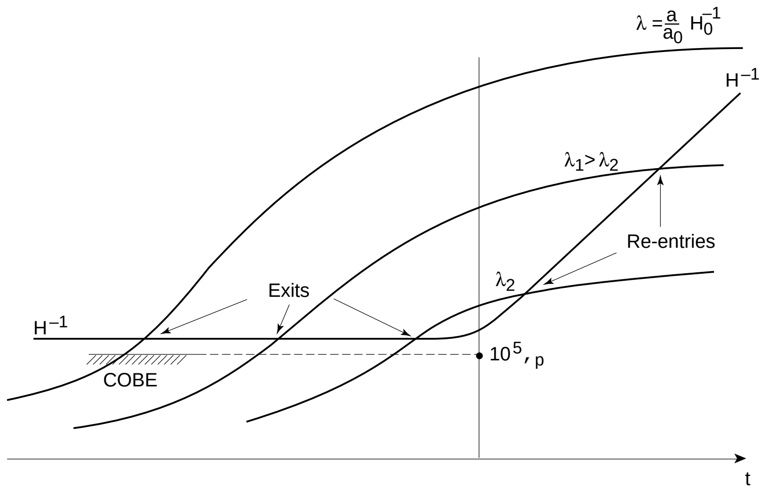

Figure 2a

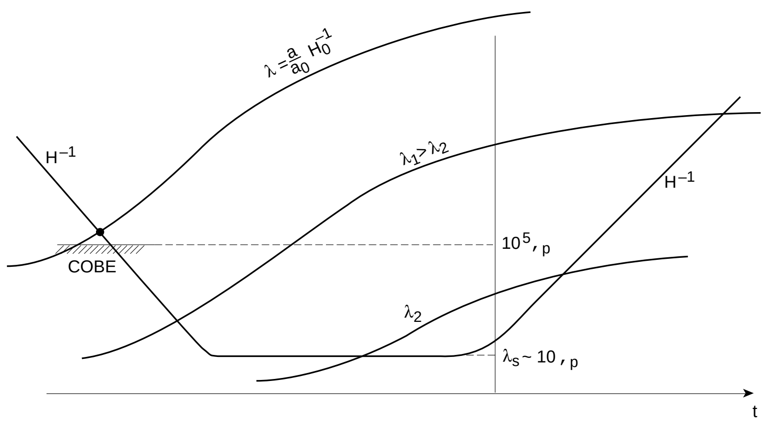

Figure 2b

4. Pre-big bang inflation as a classical instability

The key to understanding how our apparently innocent initial conditions can give inflation is in the last attribute we assumed for the primordial state: the Universe starts deeply inside the perturbative region, i.e. at very small coupling. In terms of the dilaton this means that ϕ started very large and negative. This entitles us to treat the early history of the Universe classically. Since we have also assumed that it was almost flat, we are also entitled to use the low-energy approximation to string theory. All this can be summarized by saying that we can describe the evolution of the Universe through the classical field equations of the low-energy tree-level effective action (7); in the simplest case used here for illustration, this reduces to:

$$\Gamma_{eff} = \frac{1}{2\lambda_s^2} \int d^4x \sqrt{-g}\; e^{-\phi} (\mathcal{R} + \partial_\mu \phi \partial^\mu \phi) \ . \tag{10}$$

It has been known for some time (see[12] for a review) that, if homogeneity and spatial flatness are assumed, then inflationary behaviour automatically follows from the stated initial conditions. Indeed homogeneous, spatially flat solutions fall in *four* categories, of which only *one* satisfies the pre-big bang postulate. The other three exhibit either strong coupling or strong curvature (or both) in the far past.

However, assuming homogeneity from the start is not very satisfactory. If we want to solve in a natural way the homogeneity and flatness problems, we have to start with generic (i.e. not particularly fine-tuned) initial conditions near the perturbative vacuum. During the past year this problem has been tackled by several groups, with somewhat controversial conclusions. Let me try to explain how I see the present situation.

Assume that, at some remote time much before the Big Bang, the Universe was not particularly homogeneous, in the sense that spatial gradients and time derivatives were both of the same order. Assume also, in accordance with the PBB postulate, that both kinds of derivatives were tiny in string units. It can be shown[13] that these initial conditions can lead to a chaotic version of PBB inflation since, as the system evolves, certain patches develop where time derivatives slightly dominate over spatial gradients. Provided this situation is met when the kinetic energy in the dilaton is a non-negligible fraction of the critical density, dilaton-driven inflation sets in,[13,14] blowing up the patch and making it homogeneous, isotropic and spatially flat. The evolution can be studied by analytical methods (gradient expansion[15]) since the approximation of neglecting spatial gradients w.r.t. time derivatives becomes increasingly accurate within the inflating patch.

The controversial issue is that, in order to have sufficient inflation in the patch, dilaton-driven inflation has to last sufficiently long. Its duration is not infinite since it is limited, in the past, by the conditions I just described and, in the future, by the time at which, inevitably, curvatures become of string-size and we can no longer trust the low-energy approximation. Thus, as it was actually noticed from the very beginning,[9] a successful PBB scenario does require very perturbative initial conditions, so that it takes a long time (during which the Universe inflates) to reach the BB singularity.

A particular case of this "fine-tuning" was discussed recently by M. Turner and E. Weinberg.[16] They consider a homogeneous, but not spatially flat Universe and notice that the duration of PBB inflation is limited in the past by the initial value of the spatial curvature. This has to be taken very small in string units if sufficient inflation is to be achieved.

The (almost philosophical) issue is whether this is or is not fine-tuning. String theory has a single length parameter, λ_s, but, fortunately, it has massless states and low-energy vacua (such as Minkowsky space-time) whose characteristic scale is much larger than λ_s. Hence I see nothing wrong in starting the evolution of the Universe in a state of low-energy, small curvatures, and small coupling. Actually, I find it very amusing that a classical instability pushes the Universe from low energy (curvature) and small coupling towards high energy (curvature) and large coupling.

Another result, which has emerged very recently,[13,14] is the behaviour of pre-big bang cosmology in the asymptotic past. If we evolve the system from the initial conditions I described above towards the past, we seem to find two possible behaviours. Either we reach a singularity at some finite cosmic time in the past, or we flow smoothly into a trivial space-time. The first alternative, which looks generic for positive spatial curvature (e.g. a $k = +1$ Universe with $\Omega > 1$) has to be excluded since it contradicts our basic postulate. The second alternative, which looks generic for negative spatial curvature (e.g. a $k = -1$ Universe with $\Omega < 1$), is perfectly consistent with our philosophy and leads to an interesting conjecture[14] for the whole history of time that I will describe below. It would be very interesting if pre-big bang cosmology did predict that the Universe is open, something that appears to be definitely favoured at present (see e.g.[17]) by direct measurements of the red-shift-to-distance relation and by models of large-scale-structure formation.

The complete history at which we arrive can be best drawn on a diagram (Fig. 3) reminiscent of (but actually quite different from) a Carter-Penrose diagram,[18] which artificially squashes space-time at infinity. By a simple change of the radial coordinate we can rewrite Eq. (1) as

$$ds^2 = -dt^2 + a(t)^2 \left[dR^2 + r(R)^2 d\Omega^2\right] , \qquad (11)$$

and then define the coordinates x, y in Fig. 3 in terms of t and of the proper distance aR by:

$$\tan(y \pm x) = t \pm aR . \qquad (12)$$

As $t \to -\infty$, the Universe approaches an exact vacuum of string theory. However, if equal-time hypersurfaces are taken to be those with a roughly constant energy density, we find that triviality is approached "a la Milne" i.e. the metric becomes

$$ds^2 = -dt^2 + t^2 \left[dr^2/(1 + r^2) + r^2 d\Omega^2\right] , \qquad (13)$$

while the dilaton approaches an arbitrary constant. For negative time this is just a negative curvature FRW Universe, which contracts linearly in time, $a(t) = -t$. For positive t it is the linearly expanding Universe towards which we are evolving today if $\Omega < 1$ (for a discussion of this late-time behaviour see, for instance, [19]). In the Milne

Universe the evolution of the scale factor is driven by the negative spatial curvature ($-k\ a^{-2} \to t^{-2}$). Also shown in Fig. 3 are some time-like geodesics corresponding to a fixed comoving distance R from the origin.

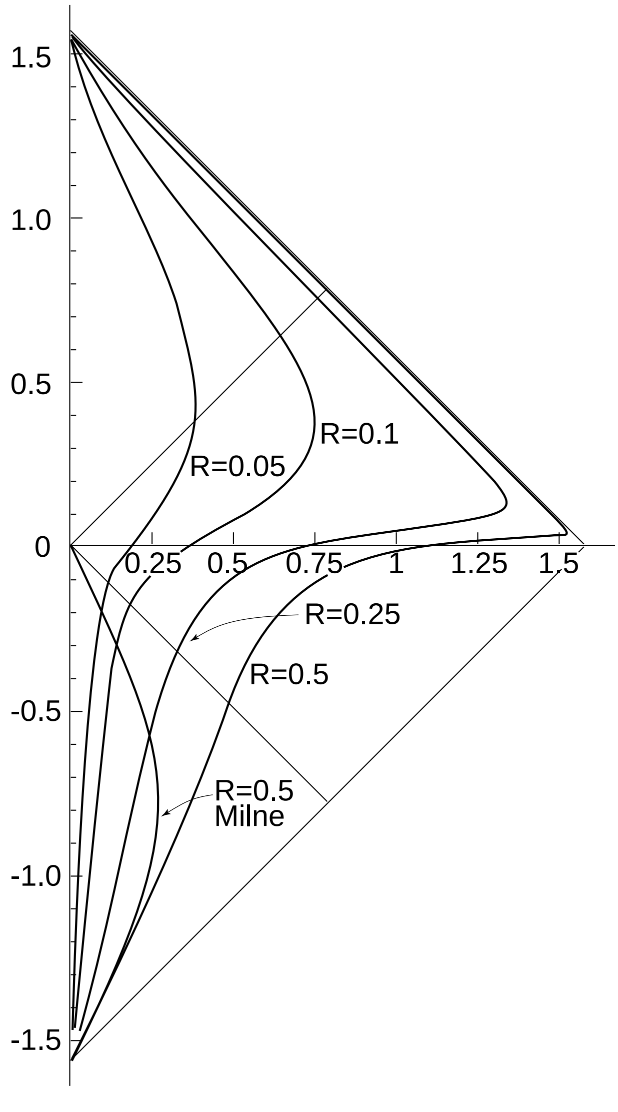

Figure 3

The generic regular solution thus approaches Milne as $t \to -\infty$ but, at any finite large $(-t)$, also contains small dilatonic (and gravitational-wave) perturbations giving $0 < \Omega \ll 1$. As $t \to -\infty$, $\Omega \to 0$. As time goes forward, instead, Ω tends to grow until, at some critical time $-T_0$, Ω becomes $O(1)$, in some region of space. From that moment on, in that "lucky" patch, the metric starts to deviate from Milne (this is shown, in particular, for the $R = 0.5$ geodesic) and dilaton-driven inflation sets in, pushing Ω extremely close to 1 in that patch.

The rest of the story goes as follows: thanks to the string UV cutoff, when the curvature becomes $O(\lambda_s^{-2})$, and/or the coupling becomes $O(1)$, a stringy mechanism prevents reaching the singularity, and a smooth transition to standard hot big bang FRW cosmology follows. An interesting quantum mechanism, described in Sec. 5, is able to provide radiation, temperature and entropy. However, by then, the inflated patch is both homogeneous and spatially flat: we have been able to produce a "good" big bang! Post-big bang evolution is from now on standard, with one qualification. Although we have achieved $\Omega = 1$ through inflation, we had to start from an open Universe and thus $\Omega = 1 - \epsilon$, $\epsilon \ll 1$. Inevitably, FRW evolution will make Ω deviate more and more from 1 until, once more, the Universe will go back to a linearly expanding Milne-like Universe. It would be wrong to think, however, that the Universe will just follow the time-reverse of its original life, firstly because the final coupling is much larger than the initial one, and, secondly, because entropy has kept increasing all the time: presumably, in this scenario, the very final stage of the Universe will consist of an ever increasingly dilute gas of slowly evaporating black holes...

5. Quantum Mechanical Heating of the Universe and Observable PBB relics

Since there are already several review lectures on this subject (e.g.[12,20]), I will limit myself to mention the most recent developments simply after recalling the basic physical mechanism underlying particle production in cosmology.[21] A cosmological (i.e. time-dependent) background coupled to a given type of (small) inhomogeneous perturbation Ψ enters the effective low-energy action in the form:

$$I = \frac{1}{2} \int d\eta \; d^3x \; S(\eta) \left[\Psi'^2 - (\nabla\Psi)^2 \right] . \tag{14}$$

Here η is the conformal time coordinate, and a prime denotes $\partial/\partial\eta$. The function $S(\eta)$ (sometimes called the "pump" field) is, for any given Ψ, a given function of the scale factor, $a(\eta)$, and of other scalar fields (four-dimensional dilaton $\phi(\eta)$, moduli $b_i(\eta)$, etc.) which may appear non-trivially in the background.

While it is clear that a constant pump field S can be reabsorbed in a rescaling of Ψ, and is thus ineffective, a time-dependent S couples non-trivially to the fluctuation and leads to the production of pairs of quanta (with opposite momenta). Looking back at Eq. (7), one can easily determine the pump fields for each one of the most interesting perturbations. The result is:

$$\begin{aligned} \text{Gravity waves, dilaton} &: \quad S = a^2 e^{-\phi} \\ \text{Heterotic gauge bosons} &: \quad S = e^{-\phi} \\ \text{Kalb-Ramond, axions} &: \quad S = a^{-2} e^{-\phi} \end{aligned} \tag{15}$$

A distinctive property of string cosmology is that the dilaton ϕ appears in some very specific way in the pump fields. The consequences of this are very interesting:

- For gravitational waves and dilatons the effect of ϕ is to slow down the behaviour of a (remember that both a and ϕ grow in the pre-big bang phase). This is the reason why those spectra are quite steep[22] and give small contributions at large scales.

- For (heterotic) gauge bosons there is no amplification of vacuum fluctuations in standard cosmology, while, in string cosmology, all the "work" is done by the dilaton. In the pre-big bang scenario, the coupling must grow by as large a factor as the one by which the Universe has inflated. This implies a very large amplification of the primordial quantum fluctuation,[23] possibly explaining the long-sought origin of seeds for the galactic magnetic fields.

- Finally, for Kalb-Ramond fields and axions, a and ϕ work in the same direction and spectra can be large even at large scales.[24] Note, incidentally, that the power of a in S is determined by the rank of the corresponding tensor. It is well known, however, that the Kalb-Ramond field can be reduced to a (pseudo)scalar field, the axion, through a duality transformation. This turns out to change S into S^{-1}, i.e. the pump field for the axion is actually $a^2 e^{\phi}$. An interesting duality of cosmological perturbations, reminiscent of electric-magnetic (or strong-weak) duality, can be argued[25] to guarantee the equivalence of the Kalb-Ramond and axion spectra.

- Many other fluctuations, which arise in generic compactifications of superstrings, have also been studied and lead to interesting spectra. For lack of time, I will refer to the existing literature.[26,27]

The possible flatness of axionic spectra in pre-big bang cosmology leads to hopes that, in such a scenario, there is a natural way to generate an interesting spectrum of large-scale fluctuations, one of the much advertised properties of the standard inflationary scenario. Work is still in progress to establish whether this hope is indeed realized.

Before closing this section, I wish to recall how one sees the very origin of the hot big bang in this scenario. One can easily estimate the total energy stored in the quantum fluctuations which were amplified by the pre-big bang backgrounds. The result is, roughly,

$$\rho_{quantum} \sim N_{eff} \; H_{max}^4 \; , \tag{16}$$

where N_{eff} is the effective number of species that are amplified and H_{max} is the maximal curvature scale reached around $t = 0$ (this formula has to be modified in case some spectra show negative slopes). We have already argued that $H_{max} \sim \lambda_s^{-1}$ and we know that, in heterotic string theory, N_{eff} is in the hundreds. Yet this rather huge energy density is very far from critical as long as the dilaton is still in the weak-coupling region, justifying our neglect of back-reaction effects. It is very tempting to assume that, precisely when the dilaton reaches a value such that $\rho_{quantum}$ is critical, the Universe will enter the radiation-dominated phase. This too is, at present, the object of active investigation.

6. Conclusion

Pre-big bang cosmology appears to have survived its first 6-7 years of life. Interest in (criticism of) it is clearly growing. It is perhaps time to make a balance sheet.

Conceptual (technical?) and phenomenological problems include:

- Graceful exit from dilaton-driven inflation to FRW cosmology is not fully understood, in spite of recent progress.[28] Possibly, new ideas borrowed from M-theory and D-branes could help in this respect.[29]
- A scale-invariant spectrum of large-scale perturbations does not look automatic, although, for the first time, thanks to the flat axion spectra, it does not look impossible either.

Attractive features include:

- No need to "invent" an inflaton, or to fine-tune potentials.
- Inflation is "natural" thanks to the duality symmetries of string cosmology.
- The initial conditions problem is decoupled from the singularity problem: a solution to the former is already shaping up and looks exciting.
- A classical gravitational instability finds a welcome use in providing inflation; a quantum instability (pair creation) is able to heat up an initially cold Universe and generate a standard hot big bang with the additional features of homogeneity, flatness and isotropy.
- Last but not least: one is dealing with a highly constrained, predictive scheme which can be tested/falsified by low-energy experiments thanks to the fact that a huge red-shift has brought the scale of Planckian physics down to that of human beings:

$$(l_P/H_0)^{1/2} \sim 1 \text{ mm} \tag{17}$$

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Note Added

Since I gave this lecture, two relevant papers have appeared:

1) N. Kaloper, A. Linde and R. Bousso (hep-th/9801073) have added further points to the criticism of the PBB scenario expressed in Ref. 16.

2) A numerical study of the spherically symmetric case by J. Maharana, E. Onofri and G. Veneziano (gr-qc/9802001) appears to support the idea discussed in Section 4 that PBB behaviour emerges generically from initial conditions sufficiently close to Milne's trivial vacuum.

These two papers confirm that much more work is still needed to clarify all the relevant issues raised by the new cosmological setup I discussed here.