Introduction:
A new ground state energy model
enabling a quantum gravity model

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(Wikipedia) **The ground state** of a quantum mechanical system is its lowest-energy state; the energy of the ground state is known as the zero-point energy of the system. An excited state is any state with energy greater than the ground state. The ground state of a quantum field theory is usually called the vacuum state or the vacuum.

state axiom of quantum mechanics: "*physical states are described by vectors of a Hilbert space, i.e. physical states are mapped injective onto the radiances of a Hilbert space*."

quantum gravity is a field of theoretical physics that seeks to describe the force of gravity according to the principles of quantum mechanics.

Mass is essentially the manifestation of the **vacuum energy**

The **Higgs boson** combines the existence of mass together with the action of the weak force. But why it provides especially to the quarks that much mass, **is still a mystery**.

"Indeed there is no observation concerned with the geometrical shape of a particle or even with an atom." ***(****E. Schrödinger)*

The conceptual idea

The central concept is about a proposed alternative harmonic quantum energy model enabling a finite "quantum fluctuation = total energy".

1. Current gravity model & its handicaps

The main characteristics of current gravity model and its related handicaps regarding the physical model requirements are

a. Metric space, affine connexions

handicaps: no scalar fields (vector fields, only), no (infinitesimal) geometry

b. differentiable manifolds

handicap: physical justification is only about continuous manifolds, additional regularity requirements are purely mathematical model driven

c. exterior differential forms, exterior product, exterior algebra

handicaps: no geometry, gravitational collapse and space-time singularities; if physical singularities in space-time are not to be permitted (R. Penrose) inside such a collapsing object at least one of the following holds

- negative local energy occurs

- Einstein's equations are violated

- the space-time manifold is incomplete

- the concept of space-time loses its meaning at very high curvatures, because of quantum phenomena

2. Current quantum model & its handicaps

The main characteristics of current quantum model and its related handicaps regarding the physical model requirements are

a. separable Hilbert space

handicap: location and momentum operator have different domains (separable Hilbert spaces) leading to non-vanishing related commutator

b. Dirac function

handicap: Dirac (delta) function regularity depends from the space dimension (due to the Sobolev embedding theorem)

3. An alternative mathematical framework

a. A separable distributional (quantum state) Hilbert space with slightly better regularity than Delta function Hilbert space (independently from space dimension n and valid for all cases n), where L(2)=H(0) test space is a closed sub-space of it

b. The standard derivative definition (momentum operator) is replaced by a Calderon-Zygmund (convolution, singular integral) PDO of order 1. In other words, Dirac's location operator is replaced by the orthogonal projection from H(-1/2) Hilbert space onto test space H(0)

c. The Dirac function concept (H(0)-inner product of a "function" and its related Fourier transform) is replaced by the inner product of an element of separable distributional Hilbert space and its related momentum. In other words, Schrödinger's momentum 0perator is replaced the orthogonal projection of the Hilbert space H(1/2) onto the test space H(0).

The proposed mathematical framework above is supposed to provide a truly infinitesimal geometry (H. Weyl). A physical interpretation could be about "rotating differentials" ("quantum fluctuations"), which corresponds mathematically to Leibniz's concept of monads. Its mathematical counterpart is the ideal points (or hyper-real numbers). This leads to non-standard analysis, whereby the number field has same cardinality than the real numbers. It is "just" the Archimedean principle which is no longer valid. This looks like a cheap prize to be paid, especially as hyper-real numbers might provide at least a proper mathematical language for the "Big Bang" initial value "function" and its related Einstein-Hilbert action functional.

Looking on hyper-real numbers from the "real" number perspective one must admit to classify the term "real" as a contraction in itself, if it is understood as *real*. Already the existence of each irrational number (not only the transcendental numbers; and the cardinality of the irrational numbers is different from the rational numbers) is ensured by an axiom, "only", i.e. the "empty space" between two rational numbers is filled with infinite irrational numbers with same cardinality as the field of real numbers itself, i.e. with multiple "universes". The difference of real numbers to hyper-real numbers is "just" the fact that there are additionally infinite small and large numbers "existent", ensured "just" by a second axiom.

PROLOG

Schrödinger E., "*Science and Humanism, Physics in our Time*", 1951,

1. The spiritual bearing of science on life: ... "*Yet the awareness that specialization is not a virtue but an unavoidable evil is gaining ground, the awareness that all specialized research has real value only in the context of the integrated totality of knowledge. The voices become fainter and fainter that accuse a man of dilettantism who dares to think and speak and write on topics that require more than special training for which he is 'licensed' or 'qualified'. And any loud barking at such attempts comes from very special quarters of two types - either very scientific or very unscientific quarters - and the reasons for the barking are in both cases translucent." ..*

3. Radical change in our ideas of matter: *... "There is the problem of matter. What is matter? How are we to picture matter in our mind? The first form of question is ludicrous. (how should we say what matter is - or, if it comes to that, what electricity is - both being phenomena given to us once only?) The second form already betrays the whole change of attitude: matter is an image in our mind - mind is thus prior to matter (notwithstanding the strange empirical dependence of my mental processes on the physical data of a certain portion of matter, viz. my brain)...*

4. Form, not substance, the fundamental concept: *... "The situation is rather disconcerting. You will ask: What are these particles then, if they are not individuals? And you may point to another kind of gradual transition, namely that between an ultimate particle and a palpable body in our environment, to which we do attribute individual sameness. ... and if the latter lack individuality, how does, say, my wrist-watch come by individuality? What is the limit? How does individuality arises at all in objects composed of non-individuals?" ...*

7. The intricacy of the continuum: *... "It seems simple to us, because the idea of the continuum seems simple to us. We have somehow lost sight of the difficulties it implies. That is due to a suitable conditioning in early childhood. Such an idea as 'all the numbers between 0 and 1' or 'all the numbers between 1 and 2' has become quite familiar to us. We just think of them geometrically as the distance of any point like P and Q from 0. ... Among the points P and Q there is also the square(2). We are told that such a number as square(2) worries Pythagoras and his school almost to exhaustion. ...There worry was highly creditable. ... The idea of a continuous range, so familiar to mathematicians in our days, is something quite exorbitant, an enormous extrapolation of what is really accessible to us." ...*

INTRODUCTION

The today´s well accepted ground state energy formula of the 1-dimensional harmonic quantum oscillator is just (!) a divergent series. Nobody seems to be concerned about this. Sophisticated renormalization techniques were developed to overcome this home made "issue", when building a quantum gravity theory. If the same nonchalance would be allowed to apply to prove the Riemann Hypothesis, this conjecture is simply proven by the only formally valid representation of Zeta(s)/(s(s-1)) as transform of the invariant self-adjoint integral operator with the "density Theta function" G(x) (EdH 10.3, 10.5).

The free energy of a system of interacting harmonic quantum oscillators to model the Planck blackbody radiation law contains same divergent series ((FeR) 10.85).

The underlying still unsolved mathematical conceptual problem is similar to the non-vanishing constant Fourier coefficient of the Theta function for the Riemann Hypothesis (RH) duality problem. The given positive answer to the RH in

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in combination with remarkable properties of the Hilbert/Riesz transforms enable an alternative mathematical ground state energy model. This finally ends up in a Hilbert space based quantum gravity model, overcoming current handicaps of (metric space, differentiable (!)) manifolds and Weyl's affine connexion concept. The latter concept leverages on Lie's concept of contact transforms (LiS).

(DeJ) 18, VII: "*The non-trivial zeros of Riemann's zeta function arise from inquiries into the distribution of prime numbers. The eigenvalues of a random Hermitian matrix arise from inquiries into the behavior of systems of subatomic particles under the laws of quantum mechanics. What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?"*

The RH solution approach with its above sketched new conception elements is also proposed as solution concept for the Yang-Mills equations with its mass gap problem.

We further mention also the paper from Aref'eva I., Ya., Volovich I. V., Quantization of the Riemann Zeta-Function and Cosmology, 2007

IN A NUTSHELL

We propose a new ground state energy model ...

- building on **Hilbert space**, alternatively to ***manifolds*** (metric space, only)

- not changing the way, "how to measure distances" (**Archimedean** axiom), but changing the "what to be measured", i.e. the structure of the underlying field from an ordered to a **non-ordered field**

- not increasing the "degree" of transcendence "complexity" (knowing that this is a question of yes/no, of course), if this is measured by Cantor's definition of cardinality (as the field of Non-Standard numbers *R does have the same cardinality than the field of real numbers R)

- applying the **Riesz and Caldéron-Zygmund Pseudo Differential Operators** (PDO) with domains in **Hilbert spaces H(-a), a>0** enabling convergent (!) quantum oscillator energy series in a Hilbert space H(-a), for appropriate a>0.The concept of affine connection is the weakest structure with which one can endow a manifold so that parallel displacement of vectors along curves is possible considerably less than a Riemannian structure is required. "Parallel displacement" is a "linear concept" along coordination (x, or y- or z-) axis; any space (or space-time) "point" does have infinitely directions of "displacements", which is nothing more than the whole "rotation" "ball" of a "mass element" d**x**, with **x**=(x,y,,z).

- enabling an infinitesimal small geometry model with an **inner product** defined by "**rotating differential forms**", alternatively to ***exterior derivatives*** based on ***differentiable (!) manifolds***

- enabling a **truly infinitesimal geometry**, alternatively to the **affine connexions** (affine, parallel infinitesimal displacements, only)

A new Hilbert scale based truly infinitesimal small geometry is provided, which is proposed to replace the Semi-Riemannian (metrical space) manifold concept (and its underlying gauge theory to enable the Standard Model of Elementary Particles) by a Hilbert space based quantum (differential forms) ground state energy model, building on an intrinsic ground state energy scalar product (of an appropriately defined Hilbert space). The latter one is consistent with existing quantum states modelling within a Hilbert space framework.

The quantization technique from quantum mechanics transfers a physical "real" world´s Partial Differential Equations (PDE) model to a mathematical (transcendental) "quantum" world Hilbert space model. The new model operates the other way around: out of the a priori "quantum/ mathematical" distributional Hilbert space world H(-a) and corresponding Pseudo-Differential Equations (PDO) (variational form representation with respect to the H(-a) inner product) it defines and re-produces the today's "real / physical" PDE world (its variational representation within a Hilbert space H(0)) by orthogonal projection from H(-a) into H(0).

Some related baseline concepts
THE "*8 fo(u)r 8*" ROAD MAP TO A QUANTUM GRAVITY

A. Eight mathematical and philosophical guides
B. Four mathematical baseline facts
C. Eight mathematical key characteristics

A. Eight mathematical and philosophical guides

The four mathematicians and the four philosophers, who provide guidance and support on our journey, are B. Riemann, J. Plemelj, H. Weyl, E. Schrödinger and G. W. Leibniz, I. Kant, A. Schopenhauer, M. Heidegger.

The baseline of Schopenhauer's philosophy concerning natural recognition is the philosophy of Kant. The common denominator of Schopenhauer's "*world as will and representation*" (="*Maya*") and Schrödinger's "*world view*" (ScE1) is the Indian philosophy of the "vedanta" (*Brahman* (cosmic soul), *Atman* (soul), *Jagart* (world)). Heidegger's philosophy adds the "human taking care" as basic expedience/purpose principle of human life/condition.

B. Four mathematical baseline facts

1. The topics "quantum gravity" and "ground state energy" are mathematical modelling challenges and not problems of physics. The purely experimental facts are given, its interpretations might be not adequate or even wrong due to not adequately applied and/or not appropriately formulated mathematical models. But the validation of physical observations and related measured data requires a common mathematical model, which does not exist existing today.
2. Physical (field) laws are formulated as Partial Differential Equations (PDE), which can be formulated in a weak form, which is about applying variational theory in a Hilbert space framework. Quantum mechanics is enabled and formulated in a specific Hilbert space framework (von Neumann, Heisenberg), which is (by purpose, by accident or by chance) the same framework, which enables probability theory. In order to define the "regularity" of the Dirac function, which is not Lebesgue integrable, it requires distributional Hilbert spaces. The Dirac function is an element of H(-n/2+e) with e>0 and n being the space-time dimension.
3. The Einstein field equations are not well-posed, non-linear PDE w/o appropriate or even missing initial or boundary value conditions. This is even valid for the Hilbert-Einstein action minimization representation of the field equations. This fact is "excused/justified/explained/anticipated" with (basically philosophical) open questions, e.g. concerning the (anyway transcendental) "appropriate" assumptions of the big-bang theory and related singularity (black whole) phenomenon. The "measurement" capability of today's (Riemannian) metric space model is built on (not only continuous (Riemannian)) differentiable manifolds, which is not appropriate to capture energy and/or action relationships between quanta to model the ground state energy phenomenon appropriately.
4. Newton's famous equation

$$m * a = F$$

has it's counterpart in Einstein's famous "tensor" field equations

$$G(i,k) = T(i,k).$$

The latter one is more exact than Newton's formula by a factor $10 * \exp(-7)$.

From a modelling perspective there is a huge conceptual difference between both equations: while Newton's equation is embedded in the Euclidean space and is purely describing the relationship between three isolated (a priori) physical terms (mass, velocity, force) as part of the Euclidean framework, the later one is describing relationships between the „geometrical" (framework) structure itself and its influencing /building matter / energies. The "geometrical" (framework) structure contains matter, which, at the same time, influences / builds / defines the framework:
the right hand side of the Einstein field equations describes the (physical) scenes and, the left hand side describes the stage itself, where the scenes happen.
This situation is not the result of an a priori defined physical requirement to be modelled or a new knowledge about physical "reality" derived from the model, but just the consequence of the chosen mathematical (tensor, manifolds, affine connexions, exterior derivatives) model.

C. Eight mathematical key characteristics

1. Given the contradictorily scene / stage framework of the Einstein field equations the probability that the underlying mathematical model can be leveraged to a well-posed physical model in any future, is 0%

2. The weak (variational) representation of PDE is based on same Hilbert space framework than quantum mechanics; the probability that physical PDE models will meet quantum mechanics models in a common variational theory framework, is 100%

3. The probability (from our human (BEING) perspective) that the universe is discrete, i.e. that there is no continuum at all, is 0%, as long as mathematics is "existing" for human BEING

4. The probability that all (physical) Nature constants are transcendental, is 100%

5. Singular integral operators (PDOs of negative order) operate on Hilbert scale. Their domain is defined per usage and required regularity. The Riesz (PD) Operators have nice properties relative to rotations ((StE), III, 1.2), which enable the definition of an inner (operator) product with domain of the linear space of differentials, which corresponds to a Hilbert space with negative scale: that enables the building of a Hilbert space of n-forms.

6. The mathematical model of a single or double layer potential V(t) induced by "matter" is given by a Lebesgue integral with Newton kernel k(s,t) and a **mass density u'(s)ds**. From a physical point of view a "potential" is a purely continuous phenomenon, but the mathematical model still require a "test particle", which is a real number (not forseen in the physical world of a potential). J. Plemelj (PlJ) proposed an alternative model of a potential, by replacing the mass density by "mass elements", i.e. to ***replace*** the **Lebesgue integral by a Stieltjes integral** and the mass density u'(s)ds **by du(s**). ..." *eine solche (mass density) Annahme erweist sich aber als eine derart folgenschwere Einschränkung, dass dadurch dem Potentiale V(t) der grösste Teil seiner Leisungsfähigkeit hinweg genommen wird*". The challenge of the (contact-interacting problem) test particle does not appear in a **distributional framework** similar to the Dirac "function" definition (LiI), (LiI1).

7. The Hilbert space L(2)=H(0) with e.g. the Hermite polynomials as orthogonal system cannot be the common denominator framework for a quantum gravity. It requires a less regular Hilbert space H(-a), with a>0 appropriately chosen (for the 1-dimensional harmonic quantum oscillator it seems to be sufficent to choose a := n/2).

In the context of distributional Hilbert spaces we refer to (LiI), (LiI1). With respect to a corresponding alternative polynomial systems to the Hermite or Laguerre polynomials we refer to (SeA). As the H(0) Hilbert space is compact embedded in H(-a) for any a>,0 (and dense with respect to the H(-a) norm) "discrete" phenomenon in H(0). like quantum leaps and uncertainty inequality, could be modelled as orthogonal projection from H(-a) into H(0).of corresponding "continuous" phenomenon in H(-a).

8. In variational theory the Dirichlet integral D(u,v) (= the Lebesgue integral of the product of gradient(u) * gradient(v)) defines the inner product of the standard (energy) Hilbert space H(1).

The Calderon-Zygmund PDO “S” operates as isomorphism from H(a+1) onto H(a), for all real “a”. The operator S is proposed to replace the gradient operator, as it has same properties as the gradient, but is defined for any Hilbert scale factor “a”.

The corresponding mathematical solution for a quantum gravity model is the following:

The Dirichlet / gradient operator, which defines the (energy) inner product D(u,v) is replaced by the Calderon-Zygmund operator to define an alternative (energy) inner product (Su,Sv)(a) with appropriately chosen scale factor "a".

It's proposed to choose the scale factor "a" in such a way, that the newly defined Calderon-Zygmund operator (energy) norm is equivalent to the norm of the Hilbert space H(0), instead of standard energy Hilbert space H(1). This goes along with a shift on the Hilbert scale by -1, i.e.

$$(Su,Sv)(-1) = (u,v)(0) =: (u,v).$$

At the same time the hyper-singular integral Su can be reformulated in variational presentation into a Stieltjes integral representation of the Riesz operators with mass element "du". The Riesz operators define isomorphism's from H(a) onto H(a). Its operator norm fulfill (Ru,Ru)=(u,u). This then enables the definition of an inner product for two "mass elements" du, dv by

$$((du,dv)) := (R(du),R(dv))(-1)= (Su,Sv)(-1) =(u,v)(0)=(u,v).$$

Due to the "regularity shift" a = -1 the Legendre transform (and therefore also the Lagrange formalism) is no longer defined in a strong sense.

The above equalities put the norm of a "mass element" into relationship to an element of the H(0) Hilbert space, which is framework of today's quantum mechanics. What remains to do is an appropriate Interpretation and probably a revisiting of the current state axiom of quantum mechanics.

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