The three Millennium problem solutions, RH, NSE, YME, and a Hilbert scale based quantum geometrodynamics

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„looking back, part (B)"

on a 10 year journey to …

3D-NSE and YME mass gap solutions coming along with a distributional Hilbert space based quantum gravity theory

The Riemann/Einstein (metric) space framework (i.e. the differentiable manifolds & affine connexions concepts) is replaced by a truly geometric Hilbert space based (quantum *state* space and related (dual) quantum *energy* space) framework.

The proposed quantum energy Hilbert space $\mathrm{H}_{1/2}$ (resp. the proposed quantum state Hilbert space $\mathrm{H}_{-1/2}$) is decomposed into $\mathrm{H}_{1/2} = H_1 \otimes H_1^{\perp}$ (resp. $\mathrm{H}_{-1/2} = H_0 \otimes H_0^{\perp}$), i.e. it is decomposed into a "kinematical" *energy* / "kinematical" action Hilbert space H_1 (resp. its related „fermions" *state* Hilbert space H_0) and its complementary "zero-point" *energy* & "zero-kinematical" action Hilbert space $H_1^{\perp}$ (resp. its related „bosons" *state* Hilbert space $H_0^{\perp}$). Mathematically speaking this is about a decomposition of the (energy) Hilbert space $\mathrm{H}_{1/2}$ into a "granular", compactly (dense) embedded Hilbert space H_1 of $\mathrm{H}_{1/2}$ and its complementary closed sub-space $H_1^{\perp}$.

Conceptually, the decompositions $\mathrm{H}_{1/2} = H_1 \otimes H_1^{\perp}$ and $\mathrm{H}_{-1/2} = H_0 \otimes H_0^{\perp}$ correspond to the "decomposition" of the field R of real numbers into rational (countable) numbers Q and irrational (non countable) numbers.

The overall physical principle in the $\mathrm{H}_{1/2}$ quantum energy Hilbert space is given by the „energy principle" (goverened by the energy conservation law). The physical principle in the „granular" (in the sense of compactly embeddedness into $\mathrm{H}_{1/2}$), kinematical quantum energy Hilbert space H_1 is given by the (original) „Leibniz least action principle", applicable for (truly) fermions (states), only.

The distributional (quantum *state*) Hilbert space $\mathrm{H}_{-1/2}$ resp. its underlying norm (i.e. with its underling "length measurements") is governed by the sum of the standard (quantum mechanics state / statistics) $\mathrm{L}_2 = \mathrm{H}_0$-Hilbert space norm and an "exponentical decay" (entropy measurements, (BrK1) note 2, (BrK6)) norm, which is weaker than any distributional "polynomial decay" norm (NiJ1). With additionally assumed regularity to the solutions of the proposed weak PDE representations, which is without any quanta theoretical physical meaning, the corresponding approximation solutions of the related classical PDE are well defined (VeW), i.e. the scalability from the "very small" quantum level to the "very large" classical level is ensured, also including now, e.g. the physical concept of "force" (based on the Lagrange formalism) or the mathematical concept of "continuity" (due to the Sobolev embedding theorem). At the same point in time H. Weyl's requirement concerning a truly infinitesimal geometry are fulfilled as well, because ...

((WeH*) p. 30): … *a truly infinitesimal geometry (wahrhafte Nahegeometrie) … should know a transfer principle for length measurements between infinitely close points only* ...

(WeH*) Weyl H., Gravitation und Elektrizität, Sitzungsberichte Akademie der Wissenschaften Berlin, 1918, 465-48.

The proposed quantum gravity model also supports the solutions of the 3D-non-linear, non-stationary (Serrin gap caused) Navier-Stokes equations problem and the Yang-Mills equations „mass" gap problem.

(B) A quantum gravity model requires some goodbyes from current postulates of quantum mechanics/dynamics models and Einstein's field model as per definition both theories are not compatible

> (KaM) p. 12: „*Because general relativity and quantum mechanics can be derived from a small set of postulates, one or more of these postulates must be wrong. The key must be to drop one of our commonsense assumptions about Nature (with respect to the underlying physical models, which are (1) continuity, (2) causality, (3) unitarity, (4) locality, (5) point particles), on which we have constructued general relativity and quantum mechanics*."

The approach of this homepage is about challenging the postulates of both theories with respect to the underlying mathematical postulated concepts. The key ingredient of quantum mechanics is the $L_2 = H_0$ Hilbert space to model quantum states with correspondingly related quantum momenta as elements of the Hilbert space H_1. The key ingredients of Einstein's field equations are Riemann's differentiable manifolds (whereby the differentiability condition is w/o any physical meaning) in combination with the concept of affine connexion (enabled by the differentiability condition) to build the metric g based (Riemann manifold) metric space (M, g). The main gap of the Einstein field equations is, that it does not fulfill Leibniz's requirement, that "there is no space, where no matter exists"; the GRT field equations provide also solutions for a vaccuum, i.e. the concept of "space-time" does not vanishes in a matter-free universe.

The proposed quantum gravity model is based on the following changes:

> (1) the Dirac „function" to model the charge of a point particle (going along with a Hilbert space $H_{-n/2-\varepsilon}$, where n denotes the space dimension, and $\varepsilon > 0$), is replaced by elements of the Hilbert space $H_{-1/2}$
>
> (2) Dirac's concept of a spin of an electron is replaced by quanta elements of the (dual) Hilbert space pair $H_{-1/2}$ (position) and $H_{1/2}$ (momentum/energy)
>
> (3) the solution of classical theoretical physics PDE is interpreted as an approximation solution to the solution of the underlying (weak) variational PDE representation of the PDE and not the other way around; from a mathematical point of view this allows reduced regularity requirements of the concerned PDE solution(s)
>
> (4) all „Nature forces" are based on the same concept of underlying „living (bosons) and kinetic (fermions) energies"; the (dual) Hilbert space pair $(H_{-1/2}, H_{1/2})$ ensures a valid Hamiltonian formalism, while the applied Lagrange formalism of the SMEP is not valid due to reduced regularity assumptions of variational solution in the above Hilbert space pair framework; however, the Lagrange formalism keeps valid for the classical approximation solutions with its underlying notions of "Nature forces". This is due to the fact that the Langrange and Hamiltonian formalisms are equivalent, if the Legendre transformation is valid, which is the case for the classical approximation solutions

With respect to (3) concerning "scalability" we quote from Smolin L., Einstein's Unfinished Revolution, The search what lies beyond the quantum, xvii:

„*In these chapters I hope to convince you that the conceptual problems and raging disagreements that have been bedeviled quantum mechanics since its inception are unsolved and unsolvable, for the simple reason that the theory is wrong. It is highly successful, but incomplete. Our task - … - must be to go beyond quantum mechanics to a desription of the world on an atomic scale that makes sense*".

The notion "force" becomes ("only") an intrinsic part on each of the considered physical situations, mathematically represented as classical PDE, which are governed by mathematical notions like "continuity" or "differentiability". The scale-up capability from weak/quantum (Hilbert space based) variational representations to e.g. continuous or differentiable function spaces is given by the Sobolev embedding theorems.

The proposed quantum gravity model in a nutshell

In the proposed quantum gravity model (supporting also the solutions of the 3D-non-linear, non-stationary Navier-Stokes equations problem and the Yang-Mills equations mass gap problem) the Riemann/Einstein (metric) space framework (differentiable manifolds & affine connexions) is replaced by a truly geometric Hilbert space based (quantum state & quantum energy space) framework. The proposed energy Hilbert space $\mathrm{H}_{1/2}$ is decomposed into an "kinematical" energy / „kinematical" action Hilbert space H_1 and its complementary "zero-point" energy & "zero-kinematical" action Hilbert space $H_1^{\perp}$; mathematically speaking this is about a decomposition of the Hilbert space $\mathrm{H}_{1/2}$ into a "granular", compactly (dense) embedded Hilbert space H_1 of $\mathrm{H}_{1/2}$ and its complementary closed sub-space $H_1^{\perp}$. Conceptually this decomposition corresponds to the "decomposition" of the field of real numbers into rational (countable) and irrational (non countable) numbers.

Current physical classical PDE model solutions are considered as approximation solutions to the underlying weak variational formulation in the proposed Hilbert space framework and not the other way around, e.g. (BrK6), (VeW). The weak variational models are governed by a common energy model concept (BrK), (BrK1), while the related "forces" phenomena become part of the specific corresponding classical PDE model, only. The distributional (quantum state) Hilbert space framework resp. its underlying norm (i.e. with its underling "length measurements") is governed by the sum of the standard (quantum mechanics / statistics) L_2 -Hilbert space norm and an "exponentical decay" (entropy measurements, (BrK1) note 2, (BrK6)) norm, which is weaker than any distributional "polynomial decay" norm (NiJ1). With additionally assumed regularity to the solutions of the proposed weak PDE representations, which is without any quanta theoretical physical meaning, the corresponding approximation solutions of the related classical PDE are well defined (VeW), i.e. the scalability from the "very small" quantum level to the "very large" classical level is ensured, also including now, e.g. the physical concept of "force" (based on the Lagrange formalism) or the mathematical concept of "continuity" (due to the Sobolev embedding theorem). At the same point in time H. Weyl's requirement concerning a truly infinitesimal geometry ((WeH) p. 30), are fulfilled as well, because ...

… a truly infinitesimal geometry (wahrhafte Nahegeometrie) … should know a transfer principle for length measurements between infinitely close points only ...

(WeH*) Weyl H., Gravitation und Elektrizität, Sitzungsberichte Akademie der Wissenschaften Berlin, 1918, 465-48.

The physical principle for the proposed kinematical Hilbert space H_1 is the (original) „Leibniz least action" principle, which is based on the „Leibniz action element" $w \cdot dt$ resp. $m \cdot v \cdot ds$ defined for any arbitrary system of arbitrary matter particles being subject to arbitrary forces. Leibniz's „actio" is defined as the action of the movement of a single matter particle during a certain time period. The least action principle in combination with Euler's variational calculus enabled multiple ODE or PDE models of physical laws, (KnA).

The „Euler-Leibniz least action" extension is based on the extension of the „Leibniz action element" $w \cdot dt$ resp. $m \cdot v \cdot ds$ to the „Hamiltonian action element" $H \cdot dt$, whereby H denotes the difference between kinetic and potential energy (KnA). In other words, applying the „Euler-Leibniz least action" extension to the kinematical H_1 Hilbert space framework would additionally include the concept of potential energy into this framework, (KnA). In ordert o avoid the „Euler-Leibniz least action" extension the required potential energy space is modelled as the complementary closed sub-space $H_1^{\perp}$ of H_1 with respect to the proposed $\mathrm{H}_{1/2} = H_1 \otimes H_1^{\perp}$ quantum energy Hilbert space norm. The corresponding quantum state Hilbert space model is then given by the distributional Hilbert space $\mathrm{H}_{-1/2}$ with its corresponding decomposition into $H_0 \otimes H_0^{\perp}$. We note that the Dirac delta distribution „function" (playing a key role in the field theory of a Dirac-electron and the Klein-Gordon equation) is an element of $\mathrm{H}_{-n/2-\varepsilon}$, $(\varepsilon > 0$, n denotes the space-dimension), i.e. even in the $n = 1$ best case the quantum states do have better regularity than the Dirac „function".

The overall physical principle in the $\mathrm{H}_{1/2}$ quantum energy Hilbert space is given by the „energy principle" (goverened by the energy conservation law). The physical principle in the compactly embedded, „granular" (in the sense of compactly embeddedness), kinematical quantum energy Hilbert space H_1 is given by the (original) „Leibniz least action principle", applicable for (truly) fermions, only. The H_1 inner product and its related norm is defined by the variational (Friedrichs) extension of the Laplace (Newton) potential operator with the corresponding domain H_1. For an approximation theory wth respect to the concept of „energy functional minimization" in a compactly embedded „approximation“ sub-space of a Hilbert space we refer to (VeW), (NiJ1). The required "time differential/variable" in the H_1 framework can be defined via the "action variable", derived as the solution of a corresponding ODE, (HeW). This concept is in line with the "thermal time hypothesis" of the loop quantum gravity (LQG), (RoC) 3.4.

The proposed quantum state Hilbert space $\mathrm{H}_{-1/2}$ with its separable and reflexive Hilbert sub-space $\mathrm{H}_0 = \mathrm{L}_2$ is also in line with a similar geometric (Hilbert space) structure as the proposed kinematical (separable Hilbert quantum-) state space in the loop quantum gravity theory, (RoC) 6.2, 6.4.2). Unfortunately, the extension of the kinematical state space to Yang-Mills fields „yields to a no longer sensible quantum state space, as this extension yields to a nonseparable Hilbert space“, (RoC) 7.2.1. Additionally the LQG requires another extension to fermions by a Grasman-valued fermion field. This „extension“ concept follows the underlying baseline concept of LQG, which is about "defining the coupled gravity + matter system by adding the terms defining the matter dynamics to the gravitational relativistic hamiltonian", ((RoC) (7.3), (7.32)).

The proposed quantum energy $\mathrm{H}_{1/2}$ decomposition concept avoids the „adding concept“ of several hamiltonians of the LQG. As a consequence, e.g. the Yang-Mills mass gap problem "just" disappears. With respect to the Einstein field equations we note that the Hilbert-Einstein action functional defined in a (differentiable) manifold framework was derived in the same way as Helmholtz derived the (classical) electro-magnetic elementary particles state equations and the equation of the second law of thermodynamics. Helmholtz’s approach is based on an extension of the „Hamiltonian least action“ principle, considering additionally also external forces. In other words, the proposed (dual) quantum state/ quantum energy Hilbert space pair $(\mathrm{H}_{-1/2}, \mathrm{H}_{1/2})$ an appropriate Hilbert space framework for an alternative Hilbert-Einstein action functional, derived from the Newton potential equation.

Plasma is the fourth state of matter, where from general relativity and quantum theory it is known that all of them are fakes resp. interim specific mathematical model items. If plasma is considered as sufficiently collisional, then it can be well-described by fluid-mechanical equations. The „bosons“ $H_1^{\perp}$ closed sub-space of the proposed energy Hilbert space $\mathrm{H}_{1/2}$ also allows an alternative modelling of plasma physics phenomena, e.g. enabling an alternative model of the "Cosmological Microwave Background Radiation" (CMBR) phenomenon with its underlying concepts of an early (or primordial) universe (where all cosmic matter was entirely ionized) as the period preceding the recombination period, where electrons and nuclei recombined and ionisation progressively decreased.

The current phase space concept can be easily adapted to the Hilbert space pairs $\overline{\mathrm{H}}_0 \coloneqq (\mathrm{H}_0, \mathrm{H}_1)$ resp. $\overline{\mathrm{H}}_{-1/2} \coloneqq (\mathrm{H}_{-1/2}, \mathrm{H}_{1/2})$ coming along with the Lebesgue integral resp. the Plemelj/Stieltjes integral concepts (BrK). The Boltzmann (statistics) entropy formula in the context of the physical phase space can be interpreted as a coarse graining entropy in the standard H_0 (state/energy) framework. The Hilbert space pair H_0 comes along with Dirac's mass density concept. It is dense in $\mathrm{H}_{-1/2}$ with respect to the $\mathrm{H}_{-1/2}$ norm, coming along with Plemelj's mass element concept; the decomposition of $\mathrm{H}_{-1/2}$ into H_0 and its complementary pair of two closed sub-spaces enables the definition of a $\mathrm{H}_{-1/2}$-based entropy definition, which can be derived from a set of axioms formulated in the separable H_0 framework.

A coarse graining entropy in the H_0 framework on the (micro/quantum) fermions granularity level can be interpreted as the tendency of (condensed H_1 energy) fermions to "move back" to the (quantum) ground state energy level of $H_1^{\perp}$. The corresponding "coarse graining entropy" in the universe can be interpreted as an alternative model for the (macro/cosmos) "expanding" universe phenomenon. The Leibniz-Einstein vision of a purely relational theory concerning the concepts of matter, space, time, event, causality etc. is modelled by (weak) variational representation of elliptic PDE in a $H_1^{\perp}$ framework, modelling a "pre-physical, ground state (zero-point) energy / zero-action world"; the step to "physical world" is triggered by the first fermion creation, modelled as orthogonal projection operator from the $H_1^{\perp}$ domain onto the granular "near distance action" fermions energy space range H_1, being accompanied by the corresponding concepts of events, causality, time and space (resp. space-time). The "physical H_1 kinematical energy world" is then governed by corresponding parabolic or hyperbolic PDE variational representations (with the "heat equation" resp. the (time-asymmetrical) "wave equation" model problems or, more precisely, the corresponding well posed Cauchy problems) in the considered Hilbert space framework (CoR). We mention that the notions "elliptic" and "hyperbolic" in the notions "elliptic PDE" resp. "hyperbolic PDE" are motivated/related by those geometric figures, while the notion "parabolic PDE" is not related to a parabola, but the a simple geometric straight line.

We mention that a mathematical analysis of physical non-linear PDEs (e.g. the kinetical Landau-Boltzmann equations) is often enabled by a valid Garding inequality, which can be interpreted as a decomposition of the non-linear operator into the sum of a linear, self-adjoint operator and a compact disturbance operator, e.g. (LiP1). With respect to the consequences of the proposed model for well-posed non-linear, non-stationary Navier-Stokes equations with correspondingly reduced regularity assumptions to the initial and boundary value functions see (B17) below.

In the following we more detail the main changes and their related impact on current gaps:

(B1) the concept of differentiable manifolds required for properly defined classical Einstein field equations needs to be avoided:

> (a) Weyl's world metric to build a „Purely infinitesimal geometry (excerpt)" is still only based on the metric space (M, g). From a mathematical point of view in order to define a geometric framework a metric space is not sufficient (the field of real numbers equipped with the distance metric is a metric space; everyone would agree that this field does not show a geometric structure). The concept of an inner product is required leading to the concept of a Hilbert space. As the related norm of an inner product is a metric, each Hilbert space is also a metric space. Our proposed Hilbert space model provides an alternative approach for a "purely infinitesimal (truly) geometry"
>
> (b) the "differentiability" requirement is without any physical meaning and even continuous manifolds would be hard to be united with a Hilbert space based quantum theory, (KaM) 1.2

(B2) functional analysis, Hilbert spaces and operators build the foundation of quantum mechanics. One famous conclusion out of it, is the Heisenberg uncertainty relationship. When applying an operator in physical models it is not all the time correctly defined as its underlying domain, which is beside the mapping the second essential part of the definition of an operator, is not specified. The standard unspoken domain assumption in quantum mechanics seems to be, that, what ever it is, it needs to fit to the "quantum state" Hilbert space model: this is the Lebesgue integral based L_2-Hilbert space, which is used e.g. in mathematical statistics and physical (Kolmogorow) turbulence and thermodynamics theory; however, the quantum mechanics model requires a Hilbert space, only

(B3) the Dirac „function" concept with its underlying space-time depending (distribution) regularity needs to be avoided just from a mathematical perspective, as well as from its sophisticated physical interpretation as a "mathematical point" particle charge; we note that when picking a real number out of the x-axis the probability that it is an irrational or even a transcendental number is 100%; this is quite an unusual measure of a physical quantity; with respect to (B1) we note that the completeness axiom required to define irrational numbers is also essential for the definition of the notion "continuity"

(B4) replacing the Dirac "function" concept by $H_{-1/2}$ distributions goes along with the definition of an inner product for differentials (BrK); the replacement can be compared with a replacement of the Archimedean ordered field of "real" numbers by the non-Archimedean ordered field of hyperreal numbers. The latter ones are also called ideal numbers, which goes back to the monadology concept of Leibniz. The term "real" is somehow missleading: every irrational number "is" its own universe, i.e. it is defined as an infinite limit of rational numbers. We note that both fields do have the same cardinality and that the Archimedean axiom basically states, that each positive real number "x" can be "measured" as product of an integer "n" times another real (standard length) number "y". Another non-Archimedean field is the Levi-Civita field

(B5) Hawking S. W., „A Brief History of Time", chapter "Elementary Particles and the Forces of Nature":

„All known particles in the universe can be divided into two groups: particles of spin ½, which make up the matter in the universe, and particles of spin 0, 1, and 2, which give rise to forces between matter particles".

"*A particle of spin 0 is like a dot: it looks the same from every direction. A particle of spin 1 is like an arrow: it looks different from different directions. Only if one turns it round a complete revolution (360 degrees) does the particle look the same. A particle of spin 2 is like a double-headed arrow: it looks the same if one turns it round half a revolution (180 degrees). Similarly, higher spin particles look the same if one turns them through smaller fractions of a complete revolution. ... there are particles that do not look the same if one turns them through just one revolution: one has to turn them through two revolutions! Such particles are said to have spin 1/2.*"

„The matter particles obey what is called Pauli's exclusion principle. … It says that two similar particles cannot exist in the same state; that is, they cannot have both the same position and the same velocity, within the limits given by the uncertainty principle. The exclusion principle is crucial because it explains why matter particles do not collapse to a state of very high density under the influence of the forces produced by the particles of spin 0, 1, and 2; if the matter particles have very nearly the same positions, they must hve different velocities, which means that they will not stay in the same position any longer. If the world had been created without the exclusion principle, quarks would not form separate, well-defined protons and neutrons. Nor would these, to gether with electrons, form separate, well-defined atoms. They would all collapse to form a roughly uniform, dense „soup"".

Mathematically speaking, the uncertainty principle is caused by different domains of the quantum position and momentum operators. (We note that an operator is only well-defined by both criteria, the mapping rule of the operator and its domain). In other words, putting both physical parameters, position and momentum, as one („Nature forces" type specific) "spin-" attribute of a corresponding particle type violates the prerequisites for well-defined position and momentum operators.

In our model Dirac's spin(1/2)-concept and its related SMEP interaction particles with spin 0, 1, and 2 are no longer required. All (energy/mass) fermions are modelled as elements of the Hilbert space H_1; the corresponding fermion states are modelled as elements of the corresponding Hilbert space H_0. The complementary sub-space $H_1^\perp$ of H_1 in $H_{1/2}$ provides a (closed sub-space) bosons model of „energy/momentum interaction elements" between fermions, replacing the three SMEP „interaction particles" model with spin 0, 1, and 2. The corresponding fermions state Hilbert space is given by H_0, while the corresponding bosons state Hilbert space is given by $H_1^\perp$, which is a closed sub-space of $H_{-1/2}$. Pauli's exclusion principle is still valid and is given implicitly, as the separable Hilbert space H_1 (the "actors") is compactly embedded into $H_{1/2}$ (the "stage"), resp. the separable Hilbert space H_0 (the "actors") is compactly embedded into $H_{-1/2}$ (the "stage"); see also (PeR) 1.3, "Phase space, and Boltzmann's defintion of entropy".

Therefore, in our model the "Nature forces" phenomena become "just" implicit part of the considered (Hamiltonian formalism based) variational representations of the considered (classical) Partial Differential Equations. We mention that the concept of a compactly embedded, sparable Hilbert space follows the same building principles and related properties, as the field of rational numbers is compactly embedded into the field of real numbers.

(B6) The discrete Shannon entropy is derived from a set of axioms showing a bunch of nice properties that it exhibite. The formally defined related "continuous" entropy based on the Riemann integral concept in (MaC) (Marsh C., Introduction to Continuous Entropy) shows several weaknesses; it "is highly problematic to the point that, on its own, it may not be an entirely useful mathematical quantity".

The current phase space concept can be easily adapted to the Hilbert space pairs $\overline{\mathrm{H}}_0 := (\mathrm{H}_0, \mathrm{H}_1)$ resp. $\overline{\mathrm{H}}_{-1/2} := (\mathrm{H}_{-1/2}, \mathrm{H}_{1/2})$ coming along with the Lebesgue integral resp. the Plemelj/Stieltjes integral concepts (BrK).

The Boltzmann (statistics) entropy formula in the context of the physical phase space can be interpreted as a coarse graining entropy in the $H_0 = L_2$ framework. The Hilbert space pair H_0 comes along with Dirac's mass density concept. It is dense in $H_{-1/2}$ (with respect to the $H_{-1/2}$ norm), coming along with Plemelj's mass element concept; the decomposition of $H_{-1/2}$ into $H_0 = L_2$ and its complementary pair of two closed sub-spaces enables the definition of a $H_{-1/2}$-based entropy definition, which can be derived from a set of axioms formulated in the separable $H_0 = L_2$ framework.

The mathematical analysis tool of the fermion Hilbert space H_1 is the Fourier transform governed by the (one-parameter) Fourier waves; the corresponding analysis tool for the complementary closed subspace of H_1 in the $H_{1/2}$ framework is the continuous (two-parameter) wavelet transform, going back to Calderón's reproducing formula for radial L_1-functions with vanishing constant Fourier term (LoA).

(B7) The geometry of the granular fermions Hilbert space H_1 (in the sense of its compactly embeddedness into $H_{1/2}$) in combination with specific properties of the Friedrichs extension of the Laplacian operator (whereby the latter defines the Newton potential) allows to distinguish between repulsive and attractive fermions:

the Friedrichs extension of the Laplacian operator is a selfadjoint, bounded operator B with domain H_1. Thus, the operator B induces a decomposition of H_1 into the direct sum of two subspaces, enabling the definition of a potential and a corresponding „grad" potential operator. Then a potential criterion defines a manifold, which represents a hyperboloid in the Hilbert space H_1 with corresponding hyperbolic and conical regions ((VaM) 11.2). This direct sum of two subspaces of H_1 is proposed as a model to distinguish between repulsive and attractive fermions

(B8) the regularity of the distribution Hilbert space $H_{-\alpha}$ containing the Dirac function is given by the condition $\alpha = n/2 + \varepsilon \ \ (\varepsilon > 0)$, where n denotes the space dimension of the underling R^n field; the Sobolev embedding theorem in the form that H_α is continuously embedded into C^0, denoting the Banach space of continuous functions, provides the linkage of the Dirac point charge concept the concept of continuity, where both notions a purely mathematical concepts (without any physical meanings on elementary quantum level) even defined resp. demanded by axioms, only; at the same point in time both concepts are used in all classical theoretical physics (Ordinary or Partial Differential Equation, ODE or PDE) model

(B9) The classical Maxwell Equations are PDE with respect to the space parameter „x" and ODE with respect to the time parameter „t". They build the foundation of Lorentz's theory of electrons. Its underlying Lorentz transformation builds the foundation of Einstein's SRT. The electric and magnetic fields in „(source) free regions", i.e. regions without charges and magnetic fields (i.e. even a Dirac point particle charge is not allowed), can travel with any shape, and will propagate at a single speed, which turned out to be light velocity c. Mathematically, the underlying hyperbolic wave equations are derived by applying the curl operator to the electric and magnetic field equations (going along with additional regularity requirements to both fields) in source free regions. Then both equations reduce to the identical vector wave equation with the single parameter c. Therefore, applying the (hyperbolic, time-symmetric) wave equation as model for gravitation waves and corresponding ODEs to „calculate back" to early universe states already anticipates that „one of the assumed nicest properties of the universe" is based on the assumption that every vacuum is source free

(B10) A variational representation of the Maxwell equations in an extended Hilbert quantum state framework $H_{-1/2}$ with source free regions in H_0 resp. H_1, only would still allow classical Maxwell and wave equation models as approximations to the "truly" quantum gravity model. However, the concepts of space, time, cause and action are only defined and valid as part of the classical PDE approximation models; the required non zero vaccuum (energy) states are element of the complementary sub space to the classical variational Hilbert spaces H_0 resp H_1. The model then would allow a correspondingly extended modified SRT including energy "quanta" into Lorentz's theory of electrons, which is claimed to overcome Einstein's mathematical problem to include gravitation forces into his (mathematically well defined) SRT

(B11) (WeH) p. 171: „On the basis of rather convincing general considerations, G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such a manner that they might possibly solve the problem of matter, by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-forth flux of energy and momentum". This concept is in line with our proposed compactly embedded „fermions" energy Hilbert space H_1 into $H_{1/2}$, where a $H_{1/2}$-based (energy) field possesses a H_1-based granular (matter) structure

(B12) the notion "symmetry" with all its mathematical (group theory, Lie groups) and physical (gauge symmetry, Higgs' symmetry break down, hidden symmetry) flavors should be replaced by the notion "self-adjointness", which is the central property of a linear operator of the Hilbert space based spectral theory in the context of the Friedrichs (self-adjoint) extension of a linear symmetric operator; a self-adjoint operator allows the definition of a related "energy" inner product /norm, (VeW)

(B13) (ChD1) pp. 1, 10-13: „*Einstein's field equations is about an unified theory of space-time and gravitations; the space-time* (M,g) *is the unknown, where M denotes a 4-dimensional manifold; one has to find an Einstein metric g, fulfilling the Einstein field equations. This is basically the equality* $G=T$*, whereby* G *denotes the Einstein tensor and* T *denotes the energy momentum tensor (e.g. the Maxwell equations). The Einstein-Vacuum equations (in the absense of matter, i.e.* $T=0$*) are given by* $R=0$*, whereby* R *denotes the Ricci tensor. Its simplest solution is the Minkowski space-time with its canonical coordinate system. Apart from Minkowski space-time it is not known, if there are any smooth, geodesically complete solution, which becomes flat at the infinity on any given spacelike direction. The main difficulties one encounters in the proof for the Cauchy Einstein-Vacuum equations with given initial data are: (1) the problem of coordinates (2) the strongly nonlinear hyperbolic features of the Einstein equations. The problem of coordinates comes along with the concept of manifolds. To write the equations in a meaningful way, one seems forced to introduce coordinates. Such coordinates seem to be necessary even to allow the formulation of well-posed Cauchy problems and a proof of a local in time existence result. Nevertheless, as the particular case of wave coordinates illustrates, the coordinates may lead, in the large, to problems of their own*."

The concept of manifolds was introduced by Riemann to model the physical phenomenon „force" as a consequence of a hyperbolic geometry, replacing Newton's concept of a „far distance force" by a „near distance force" concept. The alternative approach of this homepage is about keeping the „Riemann's formula" „force" = „geometry" ((WeH3) III, 15), but introducing a truly geometric Hilbert space framework coming along with an inner product (whereby the related Hilbert space norm defines a metric), alternatively to the current affine connected manifold framework (based on the concepts "affine connexion", "covariant derivative" and "geodesics of an affine connexion"; Schrödinger E., Space-Time Structure) to enable the definitions of a metric and an (at least) exterior product. We emphasize that the affine connexion concept is not suitable to overcome open contact body problems in the context of interaction of elementary particles

(B14) The Newton gravitation model is about the potential equation. The counterpart of the underlying Laplace operator of the potential equation in the Einstein gravitation model $G=T$ (whereby G denotes the Einstein tensor and T denotes the energy momentum tensor) is the Einstein tensor G. The weak variational formulation of the potential equation leads to the energy Hilbert space H_1. Its norm is equivalent to the L_2-norm of the gradient of the considered field. If the Newton (L_2-based variational) gravitation model is interpreted as an approximation on a more accurate $H_{-1/2}$-based variational potential equation model the corresponding potential solution can be intepreted as a compact disturbance of the Newton potential solution, which could cover all strongly nonlinear hyperbolic features of the Einstein equations enabled by "Convex Analysis in General Vector Spaces" (Zalinescu C.)

(B15) the chaotic inflation state of the early universe does not match to the second law of thermodynamics. The latter requires a permanent increase of the entropy of the universe over time, i.e. the cosmos started with an incredible low probability, but also with an incredible high ordered state, "at the same point in time" ((PeR) 2.6, "Understanding the way the Big Bang was special"). The energy/action minimizing principle is equivalent to a corresponding orthogonal projection onto a compactly embedded sub-space. This orthogonal projection can be interpreted as an extended model (symmetry → selfadjointness) of the Higgs "spontaneous symmetry break down" mass generation model. Therefore, this orthogonal projection becomes a "mass generation" operator in the sense that "mass is essentially the manifestation of the vacuum energy". In other words, there is a Hilbert space model for a perfect ordered (only vaccuum energy) system until a very unlikely first event of such a projection occured; this is because the "fermions" Hilbert (sub-) space is compactly embedded into the overall energy Hilbert space. Therefore, from a probability/ statistics theory perspective the probability of this first event is zero with respct to the underlying Lebesgue measure. It might sound sophicated or even strange, but it is just the same probability, when picking a rational number out of the field of real (including irrational and transcendental) numbers on the x-axis (which is the domain framework required to define continuous functions)

(B16) the gauge (symmetry) groups $S_3 x SU_2 x U_1$ of the SMEP (and the still missing graviton gauge group, (KaM)) could be replaced by certain self-adjoint properties of related linear operators; Fourier waves could be replaced by Calderon wavelets, while from a group theoretical perspective Calderon's wavelet and Gabor's windowed Fourier transformations are the same. They are both built by the same construction principle based on the affine-linear group resp. based on the Weyl-Heisenberg group (LoA)

(B17) when changing the variational framework from H_0 to $H_{-1/2}$ the non-linear, non-stationary Navier-Stokes equations with correspondingly reduced regularity assumptions to the initial and boundary value functions become well posed, while at the same time the Serrin gap problem disappears; from a physical modelling perspective the extended $H_{1/2}$ norm based energy measure of the non-linear term does not vanishes, in opposite to the current H_1 energy norm; at the same point in time the potential incompatibility of the initial boundary values of the NSE with the Neumann problem based prescription of the pressure at the bounding walls dissappears.

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